Investigation of Stress Intensity Factor for Corrugated Plates with Different Profiles Using Extended Finite Element (XFEM)

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***Abstract*-Corrugated plates play very important role in various engineering applications. The occurrence of crack in the body of corrugated plate might results in catastrophic failure. In the present paper there are different profiles of corrugated plates (trapezoidal, sinusoidal and triangle) that are studied. In each profile the stress intensity factor and shape factor were calculated for various crack orientations, various corrugation angles and different curvature radius for the same profile. They are all subjected to different loading conditions using Extended Finite Element Method (XFEM).**

It is found the stress intensity factor when load applied parallel to corrugation direction is higher when load applied perpendicular to corrugation direction. Also found that the stress intensity factor increase by 115% when curvature radius increases with the load applied perpendicular to corrugation. This study also found and explained that the stress intensity factor increases slightly when the corrugation angle of triangle corrugated plate increases. In all cases studied, the trapezoidal corrugated plate shows the lower values of stress intensity factor compared to the sinusoidal and triangle corrugated plates.

Keywords: Stress intensity factor, XFEM, Corrugated plate.

Nomenclature

σ: Normal stress (pa)
a : Half crack length(mm)
b :Web length (mm)
c : Flange length (mm)
Y : Shape factor
E : Young´s modulus (Pa)
ν: Poisson´s ratio
K_I: stress intensity factor (MPa$\sqrt{}$mm)

1. INTRODUCTION

The corrugated steel plate has been used in many fields of applications for a long time because of its favourable properties, for example, in the field of structure engineering, it is used as sheet pile, wall or girder web in building structures. In the field of architectural engineering, it is used for facades, roof structure, and it is basic component of composite floors [1,2]. Corrugated plate beam web is used in several applications and the probability of occurrence flaw, micro cracks, minute defect in the structure of corrugation are high under various loading and boundary conditions. So, the strength of a structure could be severely affected by the presence of crack and the defects are unavoidable in the cost effective manufacturing process. Cracks may be located or developed during the service in the zones of stress concentration and the crack may be large enough so that the crack –tip may be closer to boundary. The crack may grow to cause structure failure due to low stress which acts to a structure .This stress resulted at crack depends on the applied load and shape factor (Y) which incorporates the effect of geometry .Relation between crack length and stress intensity factor in corrugated plates for different crack orientation and loading condition is studied by Laftah [3] using Extended Finite Elements (XFEM). Laftah showed that the value of stress intensity factor and shape factor are generally increasing when crack length is increased under same loading condition. Ehsan Hedayati [4] estimated the values of stress intensity factor in oblique cracked plate using extended finite element method and finite element method. The values of SIF obtained from extended finite element method were compared with theoretical values and showed a good agreement between them with error less than 1%. Various methods have been developed over the year to simulate the creation and propagation of cracks. In conventional finite element method, the physical model to be solved is divided into a number of elements connected in a certain arrangement, usually called the "mesh ".
However when there are some internal defects, like interfaces, cracks, voids, inclusion, etc. in the domain, this will create some difficulties in the meshing process. On one hand, the element boundary must coincide with the geometric edge of the defects, which will induce some distortion on the element. On the other hand, the mesh size will depend on the geometric size of these small defects, resulting in non-uniform mesh distribution in which the meshes around the defect are very fine, while those far from defect are coarse. Moreover; defects, like cracks, will only propagate along the element edge, and not only along a natural arbitrary path. XFEM extends the piecewise polynomial function space of conventional finite element methods with extra functions. In the traditional formulation of the FEM, the existence of a crack is modeled by requiring the crack to follow element edge. In contrast, the geometry in the XFEM needs not be aligned with the element edge, which provides flexibility and versatility in modeling. The solution space is enriched by the extra enrichment functions with additional degrees of freedom at crack area while the rest of the structure modeled using ordinary FE.

Other methods, like the boundary element method and meshless method also have important contribution in solving discontinuous problems. However, some inherent flaws limit their promotion; for example, the boundary element method has drawbacks in dealing with problems of strong nonlinearity, heterogeneity, and so on. On the other hand meshless method has a lack of a solid theoretical foundation and rigorous mathematical proof, so there are some uncertain parameters like the radius of interpolation domain, background integration domain, etc,[5].

From the literature survey above, we can conclude that the XFEM has not been used for predicting the effect of crack length, plate corrugation angle and curvature radius of different configurations of corrugation on the stress intensity and shape factors. Thus, the current work is to fill this gap in this field of investigation.

2. EXTENDED FINITE ELEMENT METHOD

For the purpose of fracture analysis, the enrichment function typically consists of near –tip asymptotic function that capture the singularity around the crack tip and a discontinuous function that represents the jump in displacement across the crack surfaces has been adopted. The approximation for a displacement vector function with the partition of unity enrichment is:

$$u = \sum_{i=1}^{n} Ni(x)[u_i + H(x)a_i + \sum_{a=1}^{4} F(x) b_i^a] \qquad (1)$$

Where $N_i(x)$ are the usual nodal shape functions; u_i is the usual nodal displacement vector associated with the continuous part of the finite element solution; the second term is the product of the nodal enriched degree of freedom vector, a_i, and the associated discontinuous jump function H(x) across the crack surface; and the third term is the product of the nodal enriched degree of freedom vector; b_i^a, and the associated elastic asymptotic crack tip function, F(x). The first term on the right hand side is applicable to all the nodes in the model; the second term is valid for nodes whose shape function support is cut by crack-tip[4].

3. THEORETICAL BACKGROUND

Stress intensity factor (SIF) is the most important single parameter in fracture mechanics, which can be used to examine if a crack would propagate in a cracked structure under particular loading condition. If the stress intensity factor is known, analytic expressions for the displacement field and stress field can be solved at the crack tip location. Considering a stress element of a plane problem at crack tip location with a polar coordinate system (r , θ), the analytical solution for a stress field of a plane mode I crack at the tip location is provided by the Westergaard stress function method[6]. The stress at crack tip is shown at Fig.1 and equations, (2), (3), (4), and (5) [6].

$$\sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2} \sin\frac{3\theta}{2}\right) \qquad (2)$$

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2} \sin\frac{3\theta}{2}\right) \qquad (3)$$

In the above expression, r and θ are local polar coordinates with the origin located at the crack tip.

When θ equal to zero at the crack tip this equations become:

$$\sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}} \qquad (4)$$

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} \qquad (5)$$

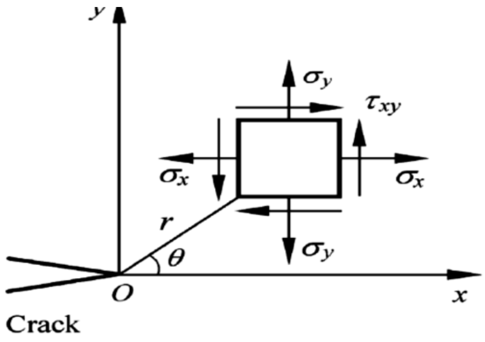

Figure1. Stress element at the crack tip

4. RESULTS AND DISCUSSIONS

The plate used in this study is carbon steel (Young Modulus E = 210GPa and Poisson´s ratio ν =0.3). A different types of corrugation trapezoidal, sinusoidal and triangle are investigated at two main directions: the first one is perpendicular to the corrugation and the second is parallel to the corrugation direction.

4.1. Flat plate

The dimensions of plate used in this case are 500mm length, 588mm width and 6mm thickness. The geometry of flat plate is shown in Fig.2.

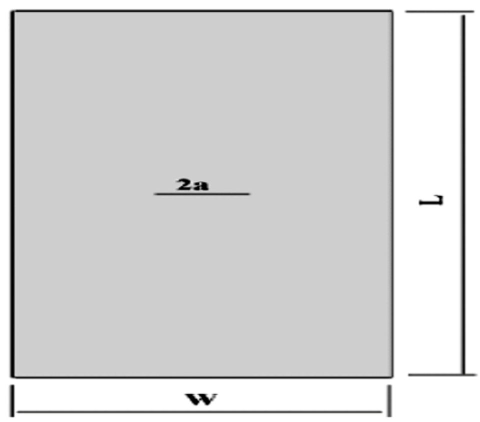

Fig.2. Geometry of flat plate

The analytical solution of stress intensity factor for central cracked plate may be found using equations 6, 7 and 8[6].

$$K_I = Y\sigma\sqrt{\pi a} \qquad (6)$$

$$Y = 1 + 0.256(a/w) - 1.152(a/w)^2 + 12.200(a/w)^3 \qquad (7)$$

$$K_n = \sigma\sqrt{\pi a} \qquad (8)$$

The analytical and numerical values of stress intensity factor are plotted against (a/w) for twelve crack length (10 to 120 mm) with step 10 mm as shown in Fig.3. This figure shows good agreement between numerical and analytical values of SIF with an error less than 2% between them. Fig.4 shows the relation between SIF and the ratio of crack length to width for flat plate and trapezoidal plate at web inclination angle 45 degree, where C45 refers to the corrugation at angle 45. This figure shows that the stress intensity factor of flat plate is higher compared with C45 due to difference in the geometry of two plates led to difference in the stiffness. The relation between crack length and shape factor is shown in Fig.5.The fringes stress distribution around crack tip is shown in Fig.6.

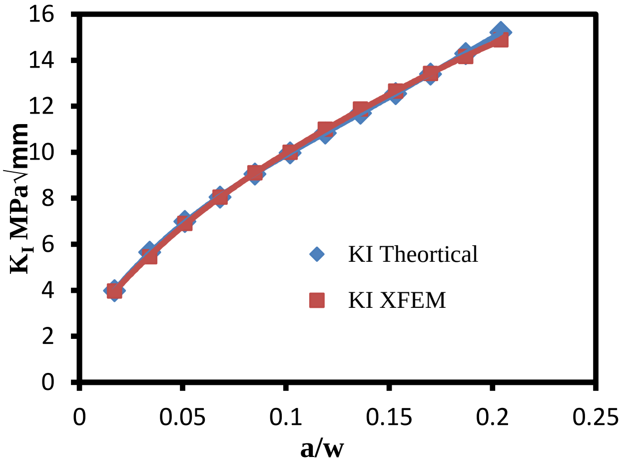

Fig. (3). Effect of crack lenght on the stress intensity

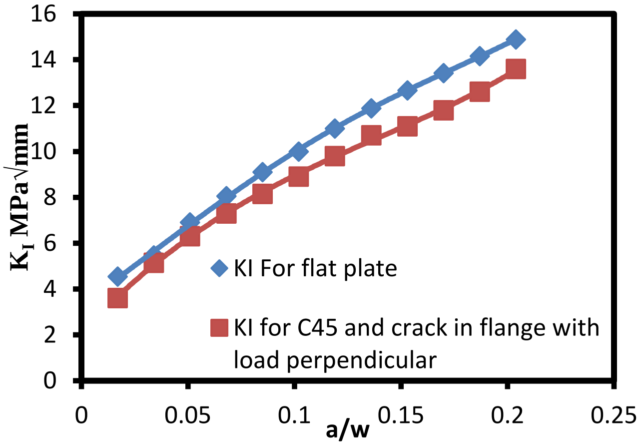

Fig.4. Effect of crack length on the stress intensity factor

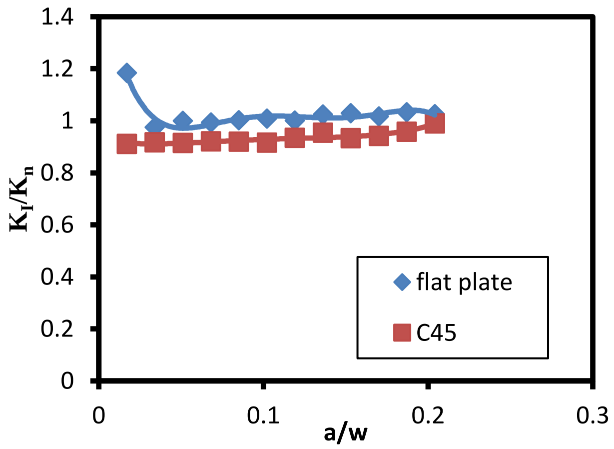

Fig.5. Effect of crack length on the shape factor

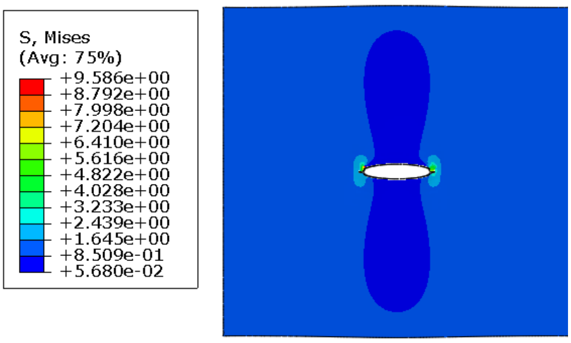

Fig.6. stress distribution fringes

4.2. Trapezoidal Corrugation plate

Trapezoidal corrugated plates composed of series of repeated (longitudinal and inclined) panels. The longitudinal panel is often called flange and the inclined is called web. Trapezoidal structure has been subjected to various crack length with different orientation and loading conditions as shown in Fig.7 will be investigated in the present work. The dimensions and geometry of trapezoidal corrugated plate is shown in Fig.8 and Table 1.

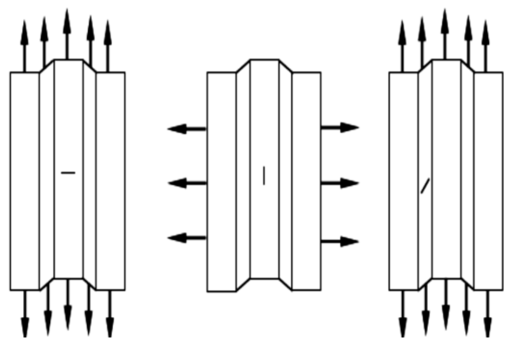

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Fig.7. Crack location and load direction for each case

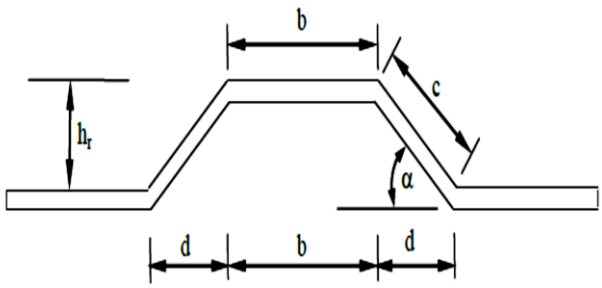

Fig.8. Geometry of corrugated plate [7]

TABEL 1. Dimensions of the trapezoidal corrugated plate.

Symbol	b	c	d	h_r	α
C30	147	147	127.3	73.5	30°
C45	147	147	104	104	45°
C60	147	147	73.5	127.3	60°

Where C30, C45, and C60 denoted the corrugation at angle 30, 45 and 60 degree. All dimensions are used in (mm). Three cases study investigated as following:

Case1: Flange central crack with load perpendicular to corrugation

A crack of different sizes is introduced to the flange of corrugated plate of different corrugation angle C30, C45 and C60 subjected to tensile load perpendicular to corrugation direction. The relation between stress intensity factor in the first mode KI and the ratio of crack length to flange width (a/b) for twelve crack length (10 to 120 mm) with step 10 mm is shown in Fig.9. The effect of corrugation geometry on the values of SIF is presented in Eq.9 obtained from the DATA FIT program with correlation factor (R^2 = 0.92).

The relation between shape factor (Y) and crack length is shown in Fig.10. From Fig. 9and Fig.10, it is shown that when crack length increases the stress intensity factor, shape factor also increases. The fringes and contours of stress distribution around the crack tip are wider spread when the corrugation angle decreases as shown in Fig.11.

$$Y\left(\frac{a}{b},\theta\right) = 1.59 + 1.49\left(\frac{a}{b}\right) - 7.41\left(\frac{a}{b}\right)^2 + 18.0044\left(\frac{a}{b}\right)^3 - 20.6545\left(\frac{a}{b}\right)^4 + 9\left(\frac{a}{b}\right)^5 - 0.0350(\theta) + 0.000398(\theta)^2 \qquad (9)$$

This equation is available for the range :

$30 \leq \theta \leq 60$

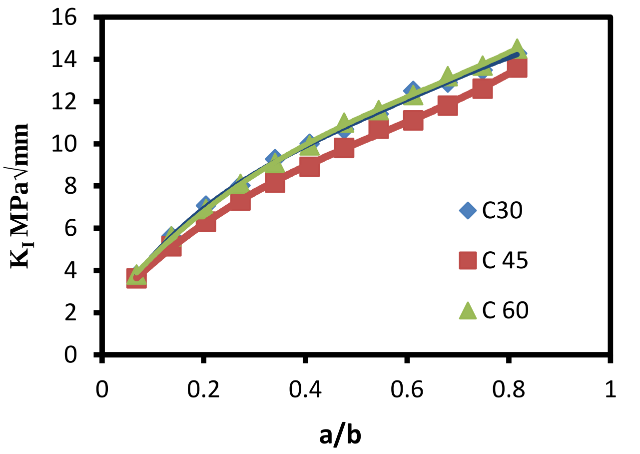

Fig. 9. Effect of crack length on the stress intensity factor K_I

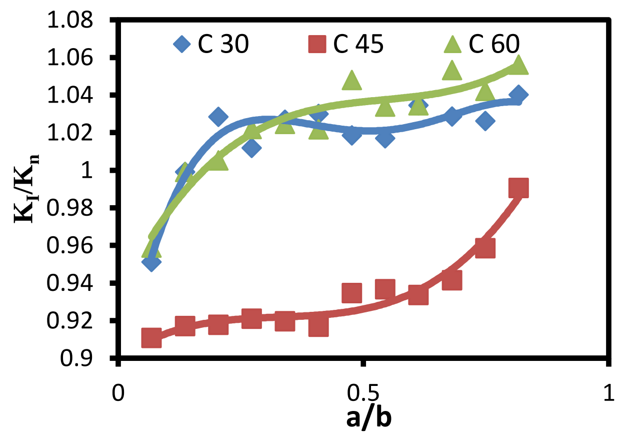

Fig. 10. Effect of crack length on shape factor

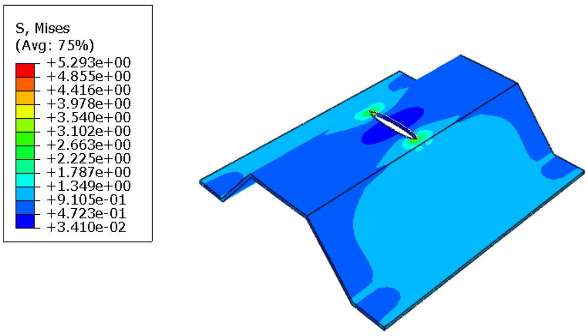

Fig. 11. Stress distribution fringes

Case 2: Flange central crack with load parallel to corrugation

In order to study the effect of crack length and corrugation geometry on the SIF and Y(a/b, θ), central crack is selected to be in flange of corrugated plate with load parallel to corrugation direction. This behavior can be explained by the relation between SIF (K_I) and the ratio of crack length to flange width (a/b) shown in Fig. 12. The effect of geometry for different corrugation angle is shown in Fig.13. The effect of corrugation geometry on the values of SIF is presented in Eq. 10 obtained from the DATA FIT program with correlation factor (R^2 = 0.93). It is clear that from Fig.12 and Fig.13, the stress intensity factor and shape factor decrease when corrugation angle increases due to corrugation stiffness. The effect of corrugation geometry is very obvious when load applied parallel to corrugation direction compared with when load applied perpendicularly. The stress distribution (fringes) around the crack tip is presented in Fig. 14.Wide fringes are noted around the crack especially at angle 30 (C 30) compared with C 60 and C 45.

$$Y\left(\frac{a}{b},\theta\right) = 2.342 - 0.12\left(\frac{a}{b}\right) + 0.63\left(\frac{a}{b}\right)^2 - 0.4353\left(\frac{a}{b}\right)^3 - 0.0547(\theta) + 0.000530(\theta)^2 \qquad (10)$$

This equation is available for the range, $30 \leq \theta \leq 60$

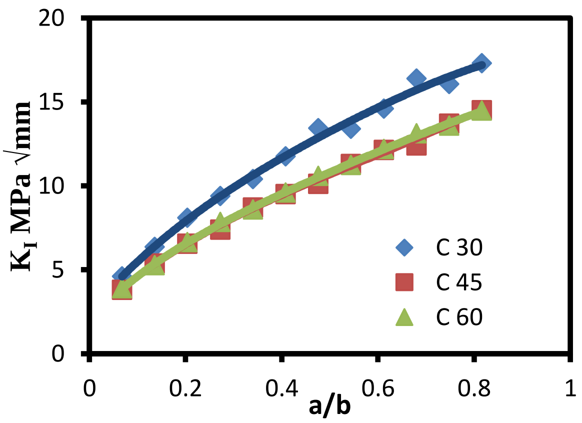

Fig.12. Effect of crack length on the stress intensity factor

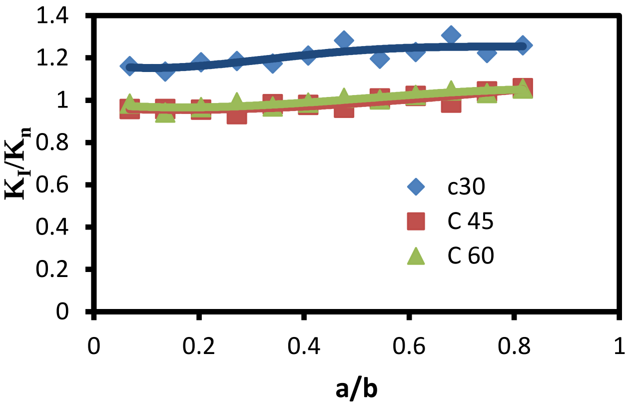

Fig13. Effect of crack length on shape factor

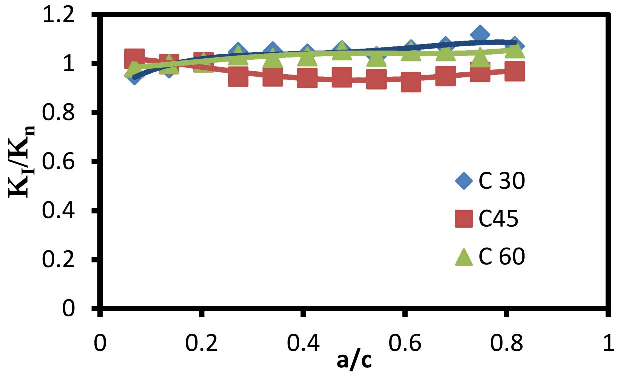

Fig. 16 .Effect of crack length on shape factor

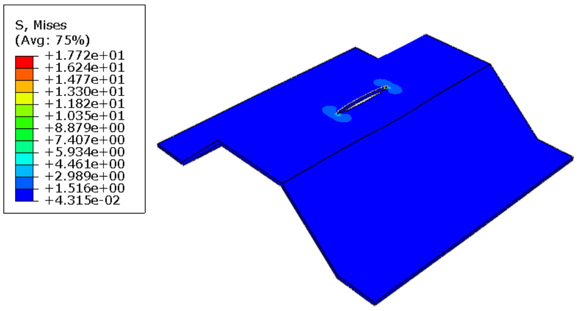

Fig. 14 Stress distribution fringes

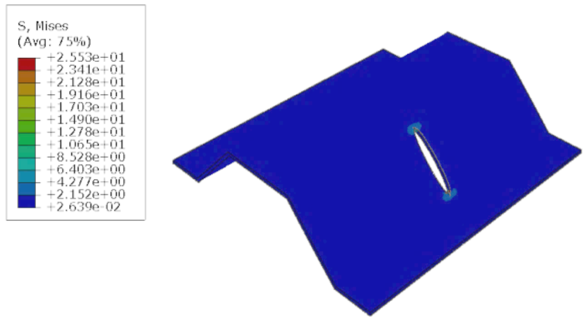

Fig. 17. Stress distribution fringes

Case3: Crack in the web with load perpendicular to corrugation

In the present case, a crack length is introduced in the web of corrugated at different angles. The plate is loaded axially and perpendicularly to the corrugation direction. Curves in Fig.15 are indicated the relation between stress intensity factor and ratio of crack length to web width (a/c). The relation between crack length and shape factor for different angle is shown in Fig.16. The effect of corrugation geometry on the values of SIF is presented in Eq. 11 obtained from the DATA FIT program with correlation factor (R^2 = 0.96). The fringes stress distribution around crack tip is shown in Fig.17.

$$Y\left(\frac{a}{c},\theta\right) = 1.0087 + 0.0574\left(\frac{a}{c}\right) - 0.01277\left(\frac{a}{c}\right)^2 - 0.0004806(\theta) \qquad (11)$$

This equation is available for the range, $30 \leq \theta \leq 60$

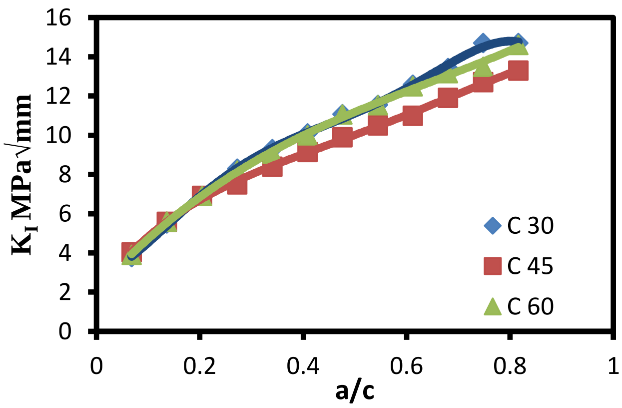

Fig. 15. Effect of crack length on the stress intensity factor

The results presented in the Fig.18 reveals that the values of stress intensity factor decrease when the corrugation angle increase. The reason for this decreasing due to the stiffness of corrugate plate will reduce and causes increase in the stress concentration around the crack tip.

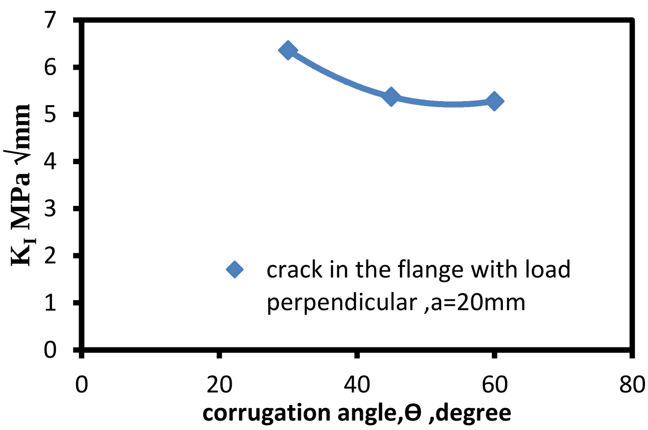

Fig. 18. Effect of corrugation angle on the stress intensity factor

4.3 Sinusoidal corrugated plate

Sinusoidal corrugated plate is the second type of the corrugation used in this study. The geometry and dimensions of sinusoidal corrugated plate used in this case is presented in Fig. 19 and Table 2. Sinusoidal plate has been subjected to various crack lengths with different orientation and loading conditions.

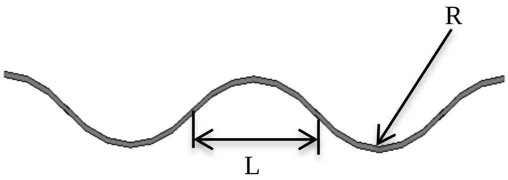

Fig.19. Geometry of sinusoidal corrugation [7]

L : length of the arch
R : radius of the arch

TABEL 2. Dimensions of triangle corrugated

Symbol	Radius	Length
CR1	127	271
CR2	295	220

Where CR1 and CR2 denoted the sinusoidal corrugation at radius 127mm and 295mm. All dimensions are used in mm. Two cases studies are investigated as following:

Case 1: Central crack in the upper wave with load perpendicular

Central crack may be selected in the upper wave with axially tension load perpendicular to corrugation direction. Fig.20 shows the relation between stress intensity factor and the ratio of crack length to the width of curvature at radius 127mm and 295mm. The effect of corrugation geometry (Y) on the stress intensity factor is shown in Fig.21and equations 12 and 13. The stress distribution around crack tip is shown in Fig. 22a-b.

$$\mathrm{Y}\left(\frac{\mathrm{a}}{\mathrm{L}}\right) = 1.0147 - 2.50\left(\frac{\mathrm{a}}{\mathrm{L}}\right) + 23.3\left(\frac{\mathrm{a}}{\mathrm{L}}\right)^2 - 76.391\left(\frac{\mathrm{a}}{\mathrm{L}}\right)^3 + 85.285\left(\frac{\mathrm{a}}{\mathrm{L}}\right)^4 \qquad (12)$$

at CR1 with correlation factor ($\mathrm{R}^2 = 0.9811$).

$$Y\left(\frac{a}{L}\right) = 1.0147 - 2.50\left(\frac{a}{L}\right) + 23.3\left(\frac{a}{L}\right)^2 - 76.391\left(\frac{a}{L}\right)^3 + 85.285\left(\frac{a}{L}\right)^4 \qquad (13)$$

at CR2 with correlation factor ($R^2 = 0.9989$).

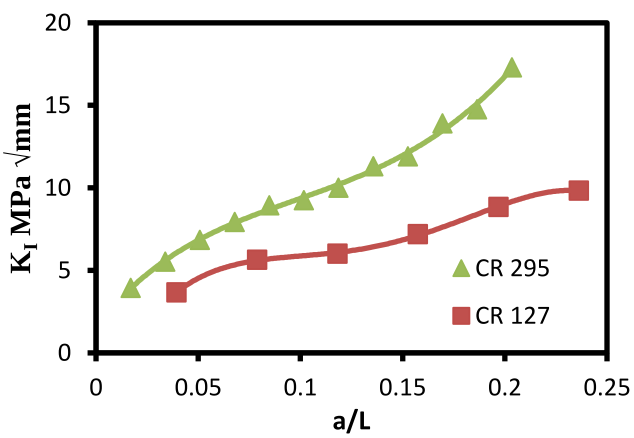

Fig. 20. Effect of crack length on the stress intensity factor

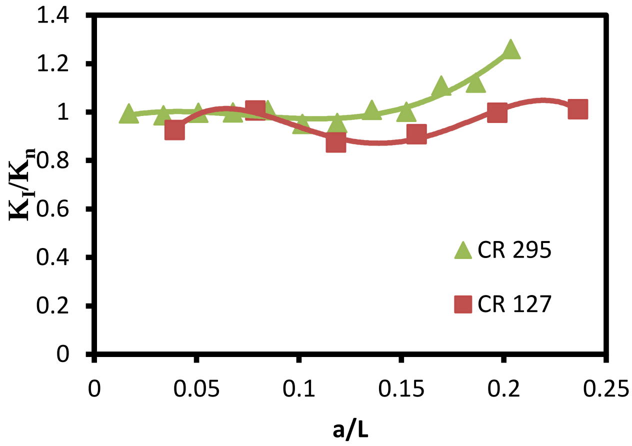

Fig. 21. Effect of crack length on shape factor

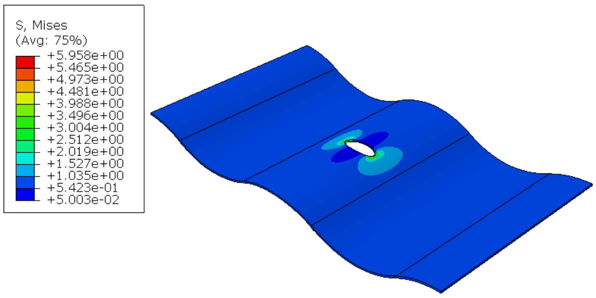

Fig. 22a Stress distribution fringes for curvature radius R 295 mm

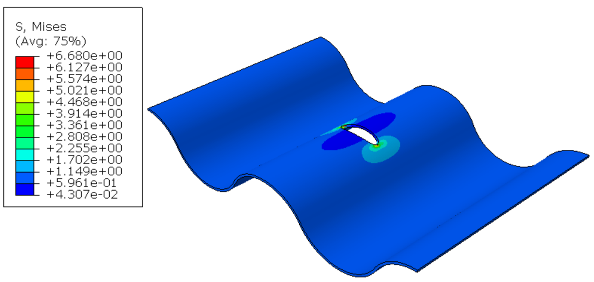

Fig. 22b Stress distribution fringes for curvature radius R 127 mm

Case 2: Central crack in the upper wave with load parallel

It is the second case of sinusoidal corrugation with two curvature radius. The corrugated plate is subjected to various crack length with load parallel to corrugation direction. Curves in Fig.23and Fig.24 show the effect of crack length on the stress intensity factor and shape factor for radius 127mm and 295mm. Equations 14 and 15 represented the shape factor Y for sinusoidal corrugation. The stress distribution fringes around crack tip are shown in Fig. 25a-b. Wide fringes are noted when corrugation radius increase due to effect of corrugation geometry.

$$Y\left(\frac{a}{L}\right) = 1.0302 - 1.2102\left(\frac{a}{L}\right) + 13.\left(\frac{a}{L}\right)^2 - 33.344\left(\frac{a}{L}\right)^3 \qquad (14)$$

at CR1 with correlation factor ($\mathrm{R}^2 = 0.9866$).

$$Y\left(\frac{a}{L}\right) = 1.1067 - 0.2947\left(\frac{a}{L}\right) + 3\left(\frac{a}{L}\right)^2 - 5.9165\left(\frac{a}{L}\right)^3 \quad (15)$$

at CR2 with correlation factor ($R^2 = 0.9917$).

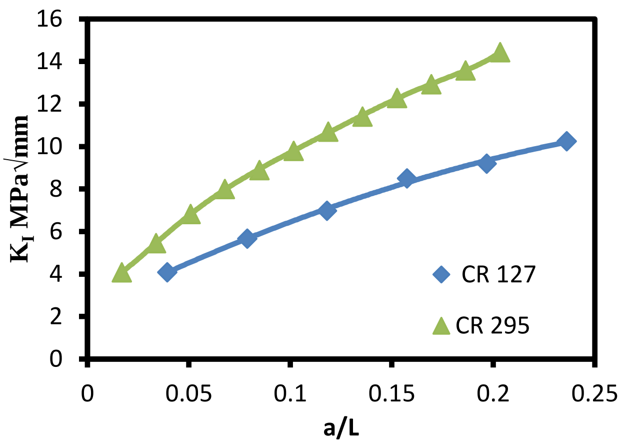

Fig. 23 Effect of crack length on the stress intensity factor

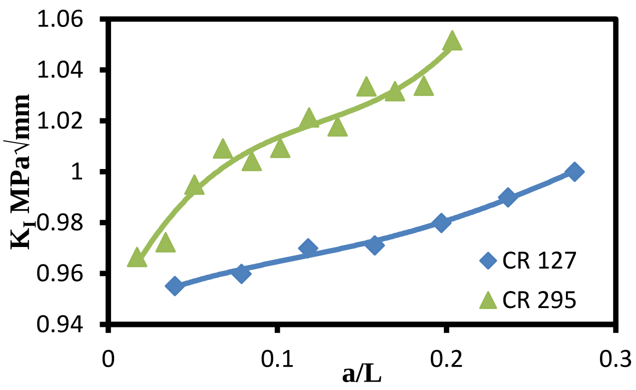

Fig. 24. Effect of crack length on shape factor

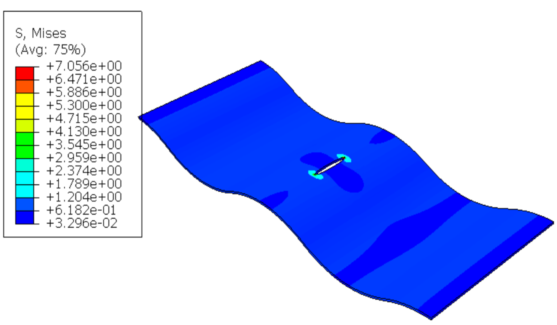

Fig. 25a Stress distribution fringes of R 295mm

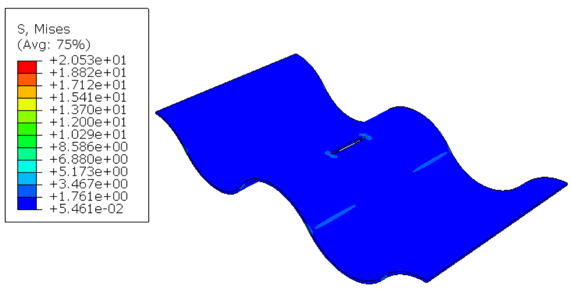

Fig. 25b Stress distribution fringes of R 127mm

4.4 Triangle corrugation plate

Triangle corrugated plate have many practical uses in wide application. Therefore, it is important to study the effect of presence of crack in the web of triangle corrugated plate at different angles (30, 45, 60) degree. The dimension and geometry of triangle corrugated plate is shown in Fig.26 and Table 3.

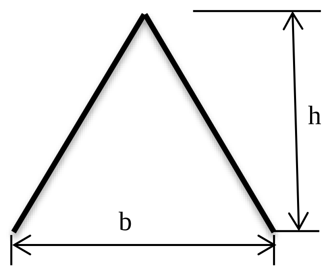

Fig. 26. Geometry of triangle corrugated plate

TABEL3. Dimensions of triangle corrugated

Symbol	b (mm)	h(mm)	θ (degree)
TC30	76	142.4	30°
TC45	112.5	135.8	45°
TC60	147	127.3	60°

Where TC30, TC45, and TC60 denoted the triangle corrugated at angle 30, 45 and 60 degree.

In order to study the effect of various crack lengths and corrugation angles on the stress intensity factor and shape factor, a crack is selected to be in the web of triangle corrugated plate with load perpendicular to corrugation direction. Fig.27 indicates the relation between the stress intensity and the ratio of crack length to the web width (a/c) and Fig.28 indicates the relation between crack length and shape factor. From Fig.29, it is shown that the triangle corrugated plate with angle of corrugation equal 60° has higher stress intensity factor compared with corrugation angles. The effect of corrugation geometry on the values of SIF presented in equations 15, 16, and 17. Stress distribution fringes show the stress contour around crack tip as shown in Fig.30.

$$Y\left(\frac{a}{b}\right) = 0.98 - 0.1537\left(\frac{a}{b}\right) + 1.823\left(\frac{a}{b}\right)^2 - 3.427\left(\frac{a}{b}\right)^3 + 1.828\left(\frac{a}{b}\right)^4 \quad (15)$$

at TC30 with correlation factor ($R^2 = 0.9962$).

$$Y\left(\frac{a}{b}\right) = 0.619 + 5.513\left(\frac{a}{b}\right) - 27.175\left(\frac{a}{b}\right)^2 + 62.966\left(\frac{a}{b}\right)^3 - 68.387\left(\frac{a}{b}\right)^4 + 28.0\left(\frac{a}{b}\right)^5 \quad (16)$$

at TC45 with correlation factor ($R^2 = 0.9702$).

$$Y\left(\frac{a}{b}\right) = 0.7828 + 2.351\left(\frac{a}{b}\right) - 6.407\left(\frac{a}{b}\right)^2 + 6.499\left(\frac{a}{b}\right)^3 - 1.733\left(\frac{a}{b}\right)^4 \quad (17)$$

at TC60 with correlation factor ($R^2 = 0.9833$).

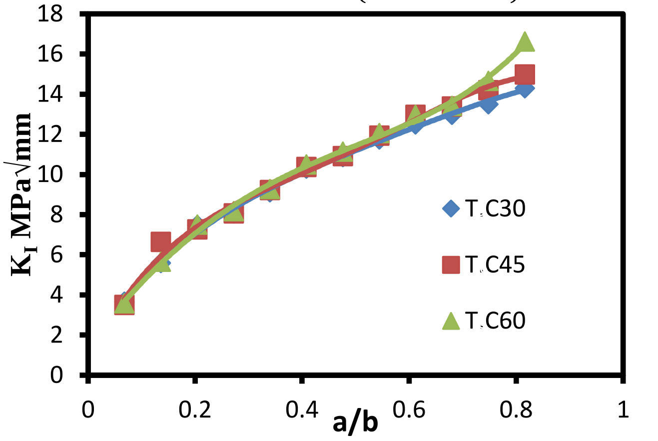

Fig. 27 . Effect of crack length on the stress intensity factor

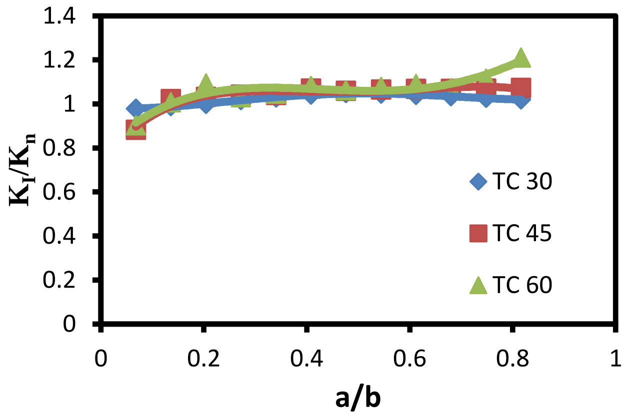

Fig28 . Effect of crack length on the shape factor.

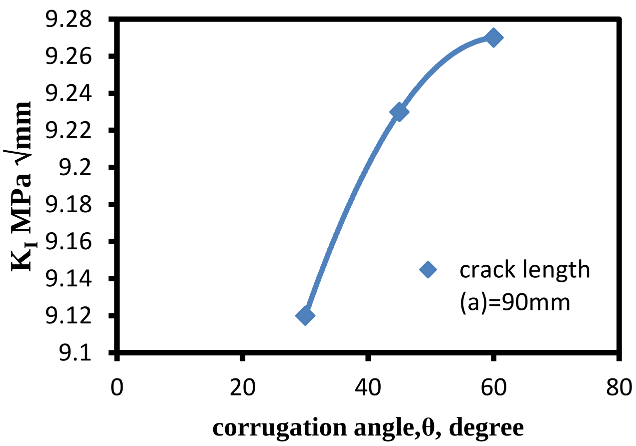

Fig29. Effect of corrugation angle on the stress intensity factor

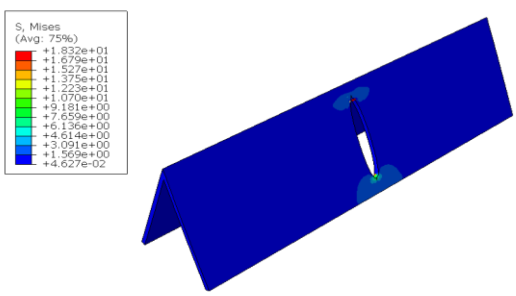

Fig. 30. Stress distribution fringes

5. CONCLUSIONS

The analysis and determination of stress intensity factor and shape factor of cracked corrugated plate under uniaxial tension is presented in this study. Three configurations of corrugated plates have been investigated: trapezoidal corrugated plate, sinusoidal corrugate plate and triangle corrugated plate. The obtained results led to the following conclusions:

1- When the trapezoidal corrugation angle decrease, the stress intensity factor increase by 120% and the stress distribution fringes are increased. The reason for this increase may be attributed to the geometry of corrugated plate approaching to the flat plate. Therefore, the stiffness of corrugate will reduce and cause increase in the stress concentration around crack tip.

2- In the case of sinusoidal corrugated plate, when the load applied parallel to the corrugation direction, the stress intensity factor and stresses around crack tip are noticed to be lower when load applied perpendicular to corrugation direction due to the difference of corrugation geometry that led to different corrugation stiffness.

3- The stress intensity factor and shape factor increase by 115% when curvature radius increases with the load applied perpendicular to corrugation.

4- In the case of triangle corrugated plate, when corrugation angle increase from 30 to 60 degree, the stress intensity factor also increase but this increase becomes obvious with increasing of the relative crack length.

5- When load applied perpendicular to the corrugation direction, the stress intensity factor of the sinusoidal corrugation for different curvature radius may be higher by 126% than the trapezoidal corrugation.

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